

Simulation of the Polarized Fermion Decays

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Abstract

In this paper the modification of the method conventionally used for the modeling of the massive fermions production and decays is proposed. The step by step algorithm is presented. Under the strict conditions the proposed method of modeling allow distinctly raise the efficiency of the computations.

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Simulation of massive fermion production (let us denote it F) with subsequent decay frequently occurs in High Energy Physics. Indeed, the angular distributions of the fermion decay products are very sensitive to its polarization.

The most obvious way is the calculation of the squared amplitude of the whole process including the fermion decays. However such way may be accompanied by some difficulties. First, the calculation of the amplitude of the whole process (including the decay chain) could be rather cumbersome. Moreover the corresponding final state can appear without the massive fermion contribution. In other words the final state of the process could be the result of the additional diagrams not containing F at all (see Fig. 1).

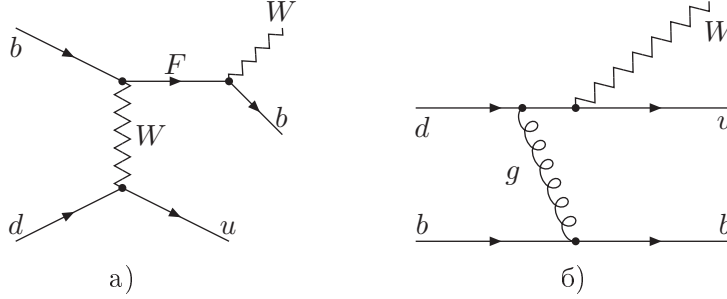


Figure 1: *The example of the diagrams describing the process with the same final state. Diagram a) corresponds to the massive fermion F resonance contribution. Here F decays into bW . Diagram b) has the same final state, but not contains fermion F .*

However, one can assume that the invariant mass of the F decay products is close to fermion mass m_F . In this case the contribution of non-resonance diagrams (see Fig.1b) will be notably suppressed.

Fairly often the problem could be simplified with the narrow resonance approximation, i.e. the intermediate fermion is considered to be «on the shell» ($p_F^2 = m_F^2$, p_F is 4-momentum of the fermion F). Such approach allows to simplify the calculations and split the simulation into two steps:

1. the production of the massive polarized fermion;
2. the subsequent decay of the polarized fermion

Thereby the squared amplitude of the process can be written in the «factorized» form:

$$|A|^2 = |A^{Prod}(s)|^2 \cdot |A^{Dec}(s)|^2 \otimes \Phi(s). \quad (1)$$

Here $A^{Prod}(s)$ and $A^{Dec}(s)$ are the amplitudes describing the production and decay of the polarized fermion, s denotes the spin of the fermion ($(sp_F) = 0$) and $\Phi(s)$ is the factor of «spin transfer» from production to decay.

The problem posed in such way used to be solved within the method of spiral amplitudes [1]. In this method the amplitudes of the fermion F production and decay calculated depending on the different helicities (λ_i). Than the total amplitude of the process can be performed as the following sum:

$$|A|^2 = \sum_{\lambda_i, \lambda_j} A^{Prod}(\lambda_i) A^{Dec}(\lambda_i) \cdot (A^{Prod}(\lambda_j))^+ (A^{Dec}(\lambda_j))^+, \quad (2)$$

The equivalent approach was proposed by Jadach and Was [2]. They showed that expression (2) can be rewritten as follows:

$$|A|^2 = |A_0^{Prod}|^2 \cdot |A_0^{Dec}|^2 (1 + \vec{H} \vec{V}). \quad (3)$$

Here A_0^{Prod} and A_0^{Dec} are the amplitudes of **unpolarized** fermion production and decay. \vec{H} and \vec{V} are so-called polarization vectors. They are determined in the fermion rest frame and contain information about fermion spin.

$$|A^{Prod}(s)|^2 = |A_0^{Prod}|^2 (1 + (Hs)) \quad (4)$$

$$|A^{Dec}(s)|^2 = |A_0^{Dec}|^2 (1 + (Vs)) \quad (5)$$

One example is given below. Accordingly to (5) the matrix element squared for the t quark decay to bW^+ with the following decay of W^+ boson to $\ell\nu$ can be written as follows:

$$|M|^2 = 2 \frac{(p_b p_\nu)(p_t p_\ell)}{(p_w^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \times \left[1 - \frac{m_t(p_\ell s)}{(p_t p_\ell)} \right], \quad (6)$$

where p_t, p_b, p_ℓ, p_ν are the momenta of the t and b quarks and final leptons, Γ_W and M_W are the total decay width and the mass of the W -boson. Hence one can derive the polarization vector for this decay

$$V^\mu = -\frac{m_t p_\ell^\mu}{(p_t p_\ell)} \quad (7)$$

As an example for expression (3) the process $u\bar{d} \rightarrow t\bar{b}$, $t \rightarrow b\ell\nu$ is considered (see Fig.2).

Using (6) the explicit expression for the matrix element squared of this process can be written in the form (compare to (3)) as follows:

$$|M|^2 \propto \left(g^4 \frac{(p_u p_b)(p_t p_{\bar{d}})}{(p_{w1}^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \right) \times \left(g^4 \frac{(p_b p_\nu)(p_t p_\ell)}{(p_{w2}^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \right) \times (1 + \vec{n}_\ell \vec{n}_{\bar{d}}), \quad (8)$$

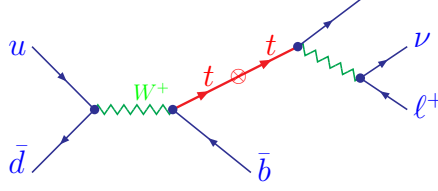


Figure 2: *Feynman diagram for the process $u\bar{d} \rightarrow b\ell\nu\bar{b}$*

where \vec{n}_ℓ and $\vec{n}_{\bar{d}}$ are directions of ℓ^+ and \bar{d} -quark momenta in t -quark rest frame.

Fairly often the expression (3) is preferable in the numerical calculations since it reduces the quantity of logical operation. Such conventional method of simulation uses discarding technique (the reject-and accept method) [3] and supposes realization of the following algorithm (see details in [2]):

- 1) to calculate the final momenta of the particles from the fermion F production process;
- 2) according to a given kinematics to evaluate the polarization vector \vec{H} in F rest frame;
- 3) to perform simulation of the F -fermion decay and to fix the decay products kinematics;
- 4) to evaluate the decay polarization vector \vec{V} (in F rest frame);
- 5) to calculate the additional weight $W = (1 + \vec{H}\vec{V})$ and using a discarding technique to reject or accept an event;
- 6) if the event is discarded than return to step 3).

However using this algorithm one can expect the increase of computation time in case of complicated expressions for the massive fermion matrix elements.

In this Note the modification of this algorithm is proposed. Very often such modification allows to raise an efficiency of the numerical calculations. The basic condition of the modified algorithm application is $|\vec{V}| = \text{const}$, i.e. the vector \vec{V} absolute value must be independent of the fermion decay simulation results.

One should notice that the value of the additional weight $W = (1 + \vec{H}\vec{V})$ (see step 5 of the algorithm) depends on the direction of \vec{V} only. Besides the different kinematics of the decay can lead to the same polarization vector \vec{V} . This ideas are the basic for our modified algorithm.

Thus we offer to choose correctly the polarization vector \vec{V} (actually the direction of \vec{V}) before the fermion decay simulation. Correctly means in order to accept an event. Hence the algorithms steps starting from step 3) modifies:

- 3as) chose the \vec{V} direction using an discarding technique with the weight $W = 1 + \vec{H}\vec{V}$;
- 4as) to simulate the massive fermion decay ;
- 5as) to evaluate the polarization vector \vec{v} corresponding to the F decay kinematics (note, that in general $\vec{v} \neq \vec{V}$ since the vectors may have different directions);
- 6as) to rotate the reference system so that vector \vec{v} coincides with \vec{V} in F rest frame .

Let us point out that for each accepted event the simulation of fermion decay made just **once**. This fact is for sure an advantage of the proposed algorithm. Note, that the method described above can be also used in case of two fermions production with a subsequent decays. In this case the additional weight takes the form:

$$W_2 = 1 + \vec{h}_1\vec{v}_1 + \vec{h}_2\vec{v}_2 + h_{ik}v_1^i v_2^k. \quad (9)$$

where (like in (3)) two vectors $h_{1,2}$ and tensor h_{ik} are determined by production kinematics. The simulation of such two fermions decays and the turning of the reference systems should be made independently.

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